

Roll No.

D-3758

M. A./M. Sc. (Final) EXAMINATION, 2020

MATHEMATICS

(Optional)

Paper Third (i)

(Graph Theory)

Time : Three Hours]

[Maximum Marks : 100

Note : Attempt any *two* parts from each question. All questions carry equal marks.

1. (a) Prove that any homomorphism is the product of a connected and a discrete homomorphism.
- (b) For two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, where $V_1 \cap V_2 = \phi$ and $E_1 \cap E_2 = \phi$, define the following :
 - (i) $G_1 + G_2$
 - (ii) $G_1 \oplus G_2$
 - (iii) $G_1 \times G_2$
 - (iv) $G_1 \wedge G_2$
 - (v) $G_1 \circ G_2$where binary operations have their usual meaning.
- (c) Write spectral properties of a graph.

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2. (a) Prove that every planar graph is k -vertex colourable iff every plane graph is k -face colourable.
(b) Prove that a critical k -chromatic graph cannot be separated by a uniquely $(k - 1)$ -vertex colourable subgraph.
(c) For any graph G , prove that $\alpha_0 + \beta_0 = n$.
3. (a) Prove that a graph is triangulated iff every minimal vertex-separator induces a complete subgraph.
(b) Prove that a graph G is a permutation graph iff G and \bar{G} are comparability graphs.
(c) Prove that every graph on $\binom{k+l}{k}$ vertices contains either a complete subgraph on $k + 1$ vertices or an independent set of $l + 1$ vertices.
4. (a) Prove that the vertex group Γ_0 and the induced edge group Γ_1 of a graph G are isomorphic iff G has at most one isolated vertex and has no component isomorphic to K_2 .
(b) Prove that if the eigen values of a graph are all distinct, then $\Gamma(G)$ is abelian and every element of Γ has order 2.
(c) Prove that the chromatic polynomial is multiplicative on the components.

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5. (a) Prove that the condensation D^* of any digraph is acyclic.
(b) Prove that the transportation network has a feasible flow iff $d(Y \cap \bar{S}) - s(X \cap \bar{S}) \leq c(S, \bar{S})$ for every subset of S of V .
(c) State and prove Kőnig's theorem.

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