Roll No. .....

D–992

M. A./M. Sc. (Fourth Semester) (Main/ATKT) EXAMINATION, May-June, 2020

MATHEMATICS

Paper Fifth

(Optional—B)

(Graph Theory—II)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. In a simple graph with n-vertices, the maximum number of edges will be :

   (a) $n - 1$

   (b) $(n + 1)$

   (c) $\frac{n(n-1)}{2}$

   (d) $\frac{n(n+1)}{2}$

2. The maximum degree of any vertex in a simple graph with n-vertices :

   (a) $n + 2$

   (b) $n - 2$

   (c) $(n + 1)$

   (d) $n - 1$

3. A vertex of degree one is called :

   (a) End vertex

   (b) Isolated vertex

   (c) Sink

   (d) None of these

4. A vertex with zero in degree is called :

   (a) Total degree

   (b) Source

   (c) Valency

   (d) None of these

5. The size of a simple graph of order n cannot exceed :

   (a) ${}^{n}C_{1}$

   (b) ${}^{n}C_{2}$

   (c) ${}^{n}C_{3}$

   (d) None of these

6. The total number of odd degree vertices of a (p, q) graph is always :
   (a) 0
   (b) 1
   (c) even
   (d) odd

7. How many vertices and edges does the graph W_n have ?
   (a) n-vertices and n edges
   (b) n-vertices and $2n$ edges
   (c) $2n$- vertices and n edges
   (d) $n + 1$ vertices and $2n$ edges

8. The rank and nullity of the complete graph K_n is :
   (a) $n - 1, \frac{1}{2}(n-1)(n-2)$
   (b) $n, \frac{1}{2}(n-1)(n+1)$
   (c) $n + 1, \frac{n(n-1)}{2}$
   (d) None of these

9. The connected planar graph having 6 vertices are 7 degrees contains regions :
   (a) 3
   (b) 5
   (c) 11
   (d) 15

10. A graph with all vertices having equal degree is known as :

   (a) Multigraph
   (b) Regular graph
   (c) Simple graph
   (d) Complete graph

11. Which of the following is true ?
   (a) Every null graph is regular of degree zero.
   (b) A complete graph K_n is regular of degree $n - 1$.
   (c) Both (a) and (b)
   (d) None of these

12. Every digraph without odd cycles has a :
   (a) No basis
   (b) 2-basis
   (c) 3-basis
   (d) 1-basis

13. Every acyclic digraph has a unique :
   (a) 4-basis
   (b) 3-basis
   (c) 2-basis
   (d) 1-basis

14. Which of the following statements is true ?
   (a) $R(s, s) \leq 2^s$
   (b) $R(s, s) \geq 2^{s+1}$
   (c) $R(s, s) \leq 2^{s-1}$
   (d) $R(s, s) \geq 2^{s-1}$

15. Which of the following is/are true ?

(a) For a digraph D.
D is not an acyclic.

(b) Every weak isograph is strong.

(c) Both (a) and (b)

(d) None of these

16. Which of the following statements is/are true ?

(a) Every unilateral digraph has a source.

(b) Every unilateral digraph has a sink.

(c) Both (a) and (b)

(d) None of these

17. Every comparability graph is :

(a) Perfect

(b) Imperfect

(c) Both (a) and (b)

(d) None of these

18. Every interval graph is :

(a) Triangulated

(b) Not necessary

(c) Both (a) and (b)

(d) None of these

19. For a graph G which is/are true ?

(a) G is a split graph

(b) G and $\overline{G}$ are triangular graph

(c) Both (a) and (b)

(d) None of these

20. Degree sequence of a graph is always :

(a) even

(b) odd

(c) Both (a) and (b)

(d) None of these

Section—B 2 each

(Very Short Answer Type Questions)

Note : Attempt all questions.

1. Explain Ramsey graph.
2. Explain permutation group.
3. Explain automorphism group.
4. Explain Bivariate coloring polynomial.
5. Explain co-chromatic graphs.
6. Explain degree sequence.
7. Explain chromatically unique graph.
8. Explain Digraph.

Section—C 3 each

(Short Answer Type Questions)

Note : Answer any *eight* questions.

Explain the following :

1. Perfectness, preserving concept.
2. Pseudosimilarity and stability.
3. Symmetry Concepts.
4. Polynomial and Graph Enumeration.
5. Interval Graphs.
6. Triangulated Graphs.

7. Perfect Graphs.
8. Covers and Basis.
9. Acyclic digraph.

Section—D 5 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. Prove that every graph on $\binom{k+l}{k}$ vertices contains either a complete subgraph on $k + 1$ vertices or an independent set of $l + 1$ vertices.

Or

Prove that for any two positive integers $S_1, S_2 \geq 2$.

2. Prove that every vertex of a composite connected graph lies on a 4-cycle.

Or

Prove that an edge transitive graph without isolated vertices is either vertex transitive or bipartite.

3. Prove that if the eigen values of the digraph D are all distinct, then Γ (D) is abelian.

Or

Prove that each cycle $C_n, n \geq 3$ is chromatically unique.

4. Prove that a weak digraph is strong iff each of its blocks is strong.

Or

State and prove Merger's theorem for digraph (vertex-form).