Roll No. ....

D–3753

M. A./M. Sc. (Previous) EXAMINATION, 2020

MATHEMATICS

Paper Third

(Topology)

Time : Three Hours] [*Maximum Marks : 100*

Note : All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Define countable and uncountable sets with examples. Show that the unit interval [0, 1] is uncountable.

   (b) Prove that :

   $$\overline{A} = A \cup D(A)$$

   where $\overline{A}$ is the closure of set A and D (A) is the derived set of set A.

   (c) Define the following with an example :

   (i) Nieghbourhood

   (ii) Limit point

   (iii) Indiscrete topology

   (iv) Co-countable topology

   (v) Base for a topology

Unit—II

2. (a) Define continuity in topological spaces. Prove that a mapping $f : X \to Y$, from a topological space X to another topological space Y is continuous if and only if the inverse image under f of every open set in Y is open in X.

(b) State and prove Lindelof theorem.

(c) State and prove Tietze extension theorem.

Unit—III

3. (a) Define disconnected and connected sets. Show that a topological space X is disconnected if and only if there exists a non-empty proper subset of X which is both open and closed in X.

(b) Define compact set. Prove that every compact subspace of a Hausdorff space is closed.

(c) Prove that a metric space X is sequentially compact if and only if it has BWP.

Unit—IV

4. (a) Define product topology. Show that the product space $X = X\{X_\lambda : \lambda \in \Lambda\}$ is Hausdorff if and only if each co-ordinate space X_λ is Hausdorff.

(b) Define para-compact space and show that every para-compact Hausdorff space is normal.

(c) State and prove Smirnov metrization theorem.

Unit—V

5. (a) Define Homotopy of paths. Show that path homotopy is an equivalence relation.

(b) Define Filter. Let A be any non-void family of subsets of a set X, then prove that there exists a filter on X containing A if and only if A has FIP.

(c) Let (X, T) be a topological space and $Y \subset X$. Then prove that Y is T-open if and only if no net in X–Y can converge to a point in Y.