Roll No. ....

D–3755

M. A./M. Sc. (Previous) EXAMINATION, 2020

MATHEMATICS

Paper Fifth

(Advance Discrete Mathematics)

Time : Three Hours] [*Maximum Marks : 100*

Note : Attempt any *two* parts from each Unit. All questions carry equal marks.

Unit—I

1. (a) State and prove fundamental theorem of semigroup homomorphism.

(b) Let (S, *) and (T, $*'$) be monoids with identities e and e', respectively. Let $f : \mathrm{S} \to \mathrm{T}$ be a homomorphism from (S,*) onto (T, $*'$). Then $f(e) = e'$.

(c) Consider the following argument for validity :

If I study, then I will not fail in mathematics.

If I do not play basket ball, then I will study.

But I failed in mathematics.

$\therefore$ I must have played basketball.

Unit—II

2. (a) Prove that two bounded lattices L_1 and L_2 are complemented if and only if $L_1 \times L_2$ is complemented.

(b) Show that dual of a lattice is a lattice.

(c) Explain the AND, OR and NOT gates and draw the logic circuit for each of the following Boolean expressions :

(i) $x \cdot y + z \cdot y'$

(ii) $ab'c + abc' + ab'c'$

Unit—III

3. (a) Define the following terms with an example :

(i) Complete Graph

(ii) Bipartite Graph

(iii) Planar Graph

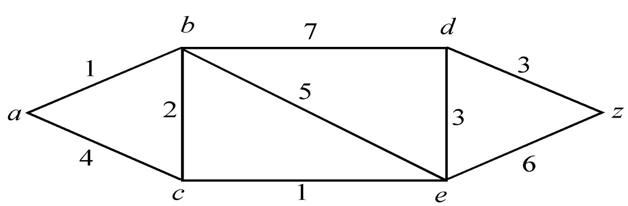
(iv) Spanning Trees

(v) Eulerian Path

(b) Show that let G be a connected planar graph with v vertices and e edges and let r be the number of regions in a planar representation of G. Then :

$$v - e + r = 2 .$$

(c) Apply Dijkstra algorithm to find shortest path from a to z in the graph given in the following figure :



Unit—IV

4. (a) Prove that for any transition function δ and for any *two* input strings x and y :

$$\delta(q_1, xy) = \delta(\delta(q, x), y) .$$

(b) Describe Moore and Mealy machines with examples.

(c) Design a finite state machine M which can add two binary numbers.

Unit—V

5. (a) Write a short note on various types of Grammar.

(b) State and prove pumping lemma.

(c) State and prove Kleene's theorem.