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M.Sc./M.A. (Previous) Examination, 2022
MATHEMATICS
PAPER FOURTH
(Complex Analysis)

Time : Three Hours]   [Maximum Marks:100

Note : All questions are compulsory. Attempt any two parts from each question. All questions carry equal marks.

Unit-1

1. (a) Derive Cauchy Integral formula.

    (b) If f(z) is analytic within and on a closed contour C except at finite number of poles and has no zeros on C, then prove that

$$\frac{1}{2\pi i}\int_C \frac{f'(z)}{f(z)}dz = N - P$$

P.T.O.

[2]

where N is the number of zeros and P the number of poles inside C.

(c) State and prove Cauchy-Goursat theorem.

Unit-2

2. (a) Evaluate : $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$ where 0 < b < a.

    (b) State and prove Riemann mapping theorem.

    (c) Prove by applying Rouche's theorem that the polynomial equation :

$$P(z) = a_0 + a_1 z + a_2 z^2 + \ldots\ldots + a_n z^n, \ a_n \neq 0 \text{ has}$$

n zeros.

Unit-3

3. (a) State and prove Weierstrass factorization theorem.

    (b) Show that the mean value property holds for harmonic functions.

    (c) State and prove Harnack's theorem.

Unit-4

4. (a) State and prove Poisson-Jensen formula.

    (b) State and prove Hadamard's factorization theorem.

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(c) Find the order of the following functions:

(i) e^{e^z}

(ii) $\cos \sqrt{2}$

Unit-5

5. (a) State and prove Bloch's theorem.

(b) State and prove Little Picard theorem.

(c) Prove that, if f is an entire function that omits two values, then f is a constant.